SySc 512 – Quantitative Methods of Systems Science

1. Dynamics

Homework 1: Dynamical Systems: Pictures and Maps.

(1) Derive a general solution for the differential equation:

$$\dot{x} = -0.1x$$

Plot x as a function of t for boundary conditions, $x_0 = \pm 1, \pm 2$.

(2) Plot graphs for the following two systems;

(a) The van der Pol oscillator:

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x + y(1 - x^2) \end{aligned}$$

(b) The two-eyed monster:

$$\begin{aligned} \dot{x} &= y + y^2 \\ \dot{y} &= -\frac{1}{2}x + \frac{1}{5}y - xy + \frac{6}{5}y^2 \end{aligned}$$

Plot both x and y versus time, and x versus y.

(3) Plot a graph of the Hénon map (where $\alpha > 0$ and $|\beta| < 1$):

$$\begin{aligned} x_{n+1} &= 1 - \alpha x_n^2 + y_n \\ y_{n+1} &= \beta x_n \end{aligned}$$

Plot x and y for $\alpha = 1.2$ and $\beta = 0.4$.

(a) Show that the Hénon map undergoes a bifurcation from period-one to peroid-two behavoir exactly when $\alpha = \frac{3(\beta-1)^2}{4}$ for fixed β.

(b) Investigate the bifurcation diagrams of the Hénon map by plotting the x_n values as a function of α for $\beta = 0.4$

You may use the Matlab code from Lynch that is posted on the class website.