# SySc 512 – Quantitative Methods of Systems Science

**Homework 2: Dynamical Systems: Linearization and Projection.**

(1) Use the following variant of the van der Pol system for this problem:

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= -x + (0.2 - x^2)y\end{aligned}$$

(a) Writing the differential equation for a continuous time dynamical system

$$\dot{\mathbf{x}} = F(\mathbf{x})$$

where both $\mathbf{x}$ and $F(\mathbf{x})$ are vectors, a point $\bar{\mathbf{x}}$ is called a *fixed point* or *equilibrium* if $F(\bar{\mathbf{x}}) = 0$. At each fixed point of the van der Pol systems find the matrix of partial derivatives with components

$$A_{i,j} = \left.\frac{\partial\,(F(\mathbf{x}))_i}{\partial \mathbf{x}_j}\right|_{\mathbf{x}=\bar{\mathbf{x}}}$$

(b) For each matrix in (1a) find the eigenvalues and eigenvectors.

(c) Let

$$x(0) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

and plot solution trajectories $\{x(t) : t \in [0, 25]\}$ for the two cases:

$$\begin{aligned}\dot{x} &= F(x) \text{ Where } F \text{ is given by the van der Pol equations} \\ \dot{x} &= Ax \text{ Where } A_{i,j} = \left.\frac{\partial(F(x))_i}{\partial x_j}\right|_{x=[0,0]^T}\end{aligned}$$

(2) Show that the one-parameter system

$$\begin{aligned}\dot{x} &= y + \mu x - xy^2 \\ \dot{y} &= \mu y - x - y^3\end{aligned}$$

undergoes a Hopf bifurcation at $\mu_0 = 0$. Plot three phase portraits for $\mu < 0$, $\mu = 0$, and $\mu > 0$.

(3) For the 3-d Rössler system plot the 3-dimensional phase portrait and 2-d projections of the attractors.