SYSC 512 Quantitative Methods of Systems Science

Portland State University
Credits: 4
Instructor: Patrick Roberts

Time and Location
Mon. & Wed. 4:00-5:50
Harder House 104

Patrick Roberts: robertpa@ohsu.edu
Office: HH 03, Mon & Wed 1:00 PM -2:00 PM.

00_SYSC512.psd

ScSy 512: Quantitative Methods of Systems Science

Class outline: (20 classes during 10 weeks.)

Class lectures will present examples of how quantitative methods are applied to systems.
Homework excercises will focus on mathematical techniques.

Part I: Dynamics. (5 classes)
Key Ideas: Non-linear differential equations, continuous and discrete;
Linearization, eigenvalues and eigenvectors; Stability and Lyapunov functions.

Part II: Optimization. (5 classes)
Key Ideas: First and second order conditions, Lagrange multipliers, variational calculus.

Part III: Uncertainty. (5 classes)
Key Ideas: Distributions for discrete and continuous random variables.
Bayes rule. Estimation and bias variance trade-off.

Software: The numerical exercises can be solved using your favorite software,
but the *supported* package will be **Matlab**. PSU has a site license for on-campus use.
Octive is a free software package that uses a syntax similar to Matlab.

01_outline.psd

Desired outcomes:

1. To help students understand how quantitative methods can be used to analyze, explain, predict, and control the behavior of systems.

2. To provide students with hands-on experience using current research tools to investigate the concepts underlying these quantitative methods.

3. To help students to develop an intuition for the dynamics of simple and complex sytems.

02_outcomes.psd

Evaluation:

1. Weekly homework: exercises using quantitative techniques (scored 0-1).

2. Exams, midterm and final (scored on a 0-4 scale).
   Final grade: 1/3 homework, 2/3 exams

Text: There will be 3 texts, one for each section of the course.

- Dynamical Systems with Applications using MATLAB (2004) Stephen Lynch
- Optimization Theory with Applications (1987) Donald A. Pierre
- Probability Theory: A Concise Course (1977) Y.A. Rozanov

I will also provide the sources I use for the lectures, and suggest further reading.

Class Website:
http://www.sysc.pdx.edu/classes/sysc512/

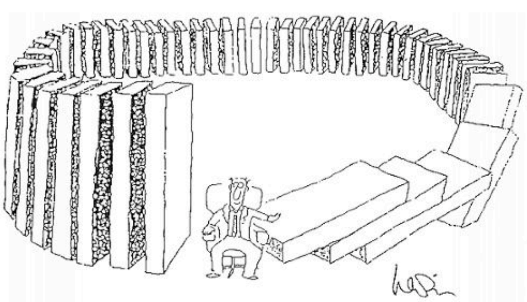
03_evaluation.psd

Why quantify systems?

To understand
To predict
To control
To manage

04_why.png

Quantification: An important tool for research

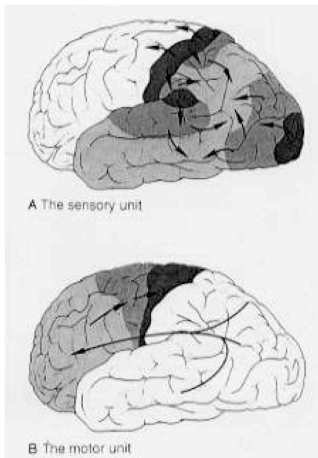


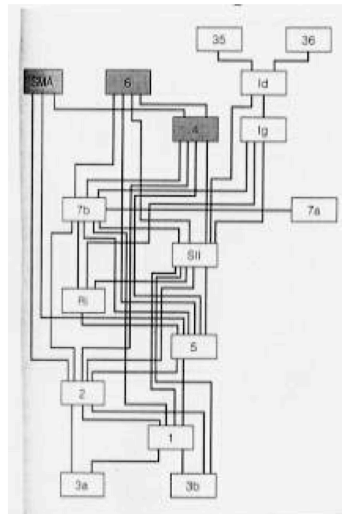
- Enhances our understanding
- Generates new hypotheses about interactions
- Allows us to test hypotheses
- Allows us to simulate models

05_quantify.png

A tool for understanding complex systems: Higher-order brain function

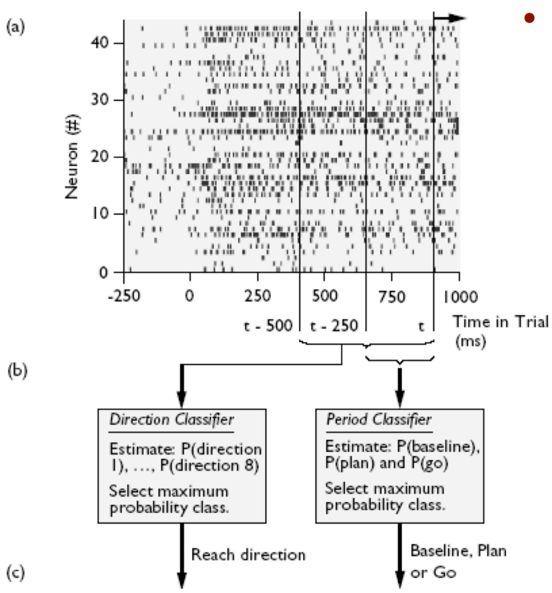
- Provides building blocks for biologically relevant neural network research

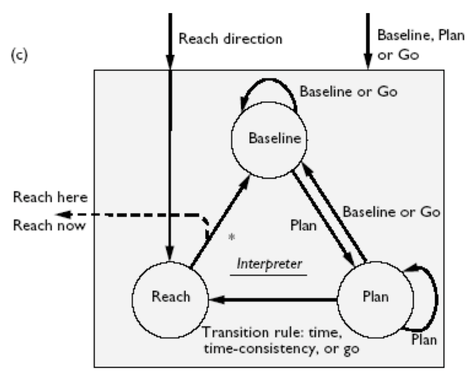




06_understand.png

- Helps us to predict functional patterns of neuronal firing





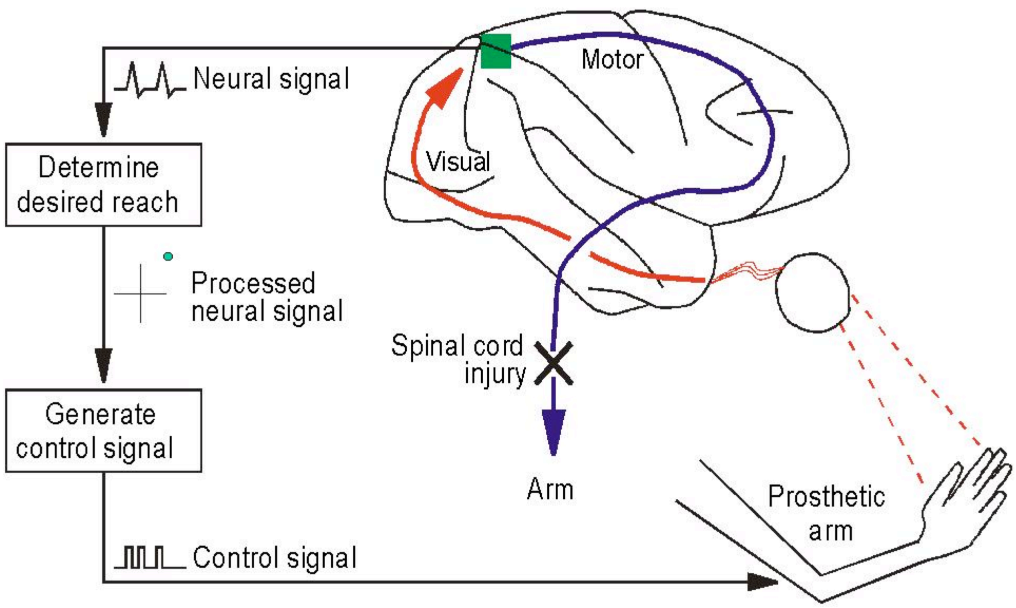
*This transition causes a high-level,cognitive control signal to be issued stating: reach here (from direction classifier), reach now.

Fig. 2. Computational architecture for generating control signals from PRR plan activity. **(a)** Spike rasters from one trial for each of 41 PRR neurons. The goal visual target occurs at 0 ms. The onset of arm movement occurs after 1100 ms (not shown). **(b)** Classifiers use neural activity from fixed-width sliding analysis windows to estimate the direction of arm movement (direction classifier) and the current neural/behavioral period (period classifier). **(c)** The interpreter receives the stream of period classifications (i.e., baseline, plan or go) from the period classifier and the stream of movement direction classifications (e.g. downward reach) from the direction classifier. The interpreter consists of a finite state machine that transitions among three states (baseline, plan and reach) according to the period classification at each time step.

Neural prosthetic control signals from plan activity, Shenoy et. al., NeuroReport, Vol 14(4), 2003.

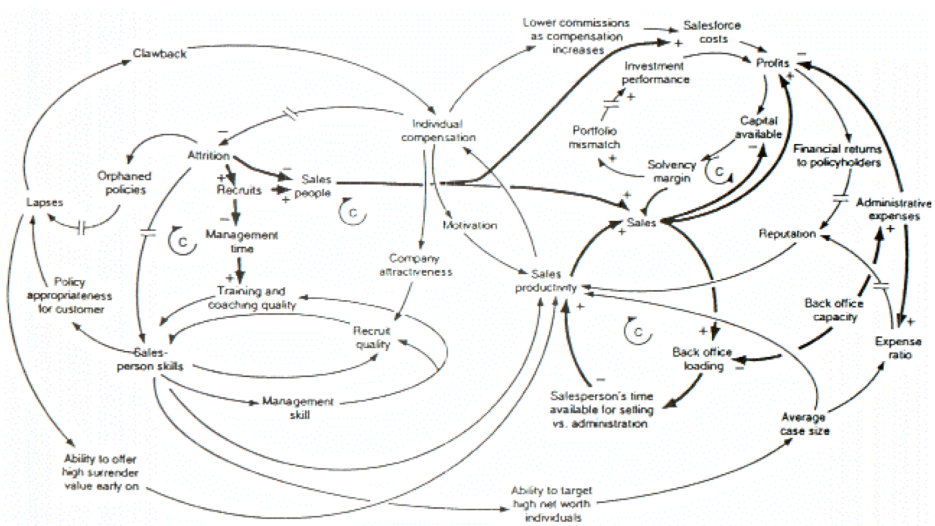
07_predict.png

A tool for neuroprosthetic device research



08_control.png

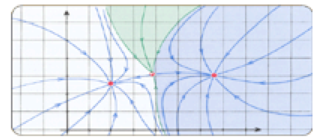
Manage and Improve Complex Systems
Without Breaking Them



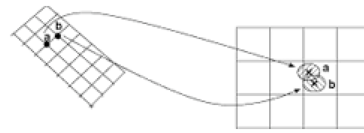
09_manage.png

Part 1: Dynamics

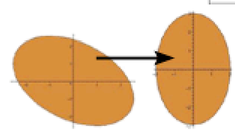
Apr 2 (01) **2-Dimensional flow geometries**. HW1



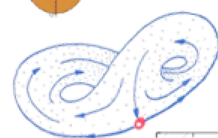
Apr 4 (02) **Discrete dynamics & Mappings**.



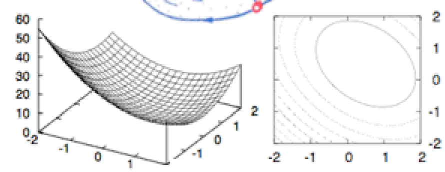
Apr 9 (03) **Diagonalization & eigenvalues**. HW2



Apr 11 (04) **Higher dimensional dynamics & linearization**.



Apr 16 (05) **Stability & Gradient systems**. HW3

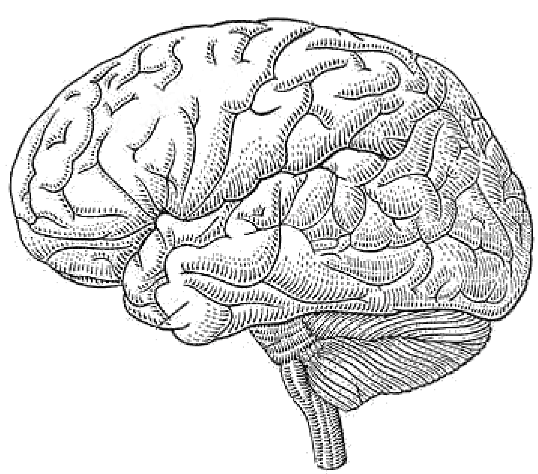


<u>Dynamical Systems with Applications using MATLAB</u> (2004) Stephen Lynch

10_Dynamics.psd

Example 1: The Human Brain

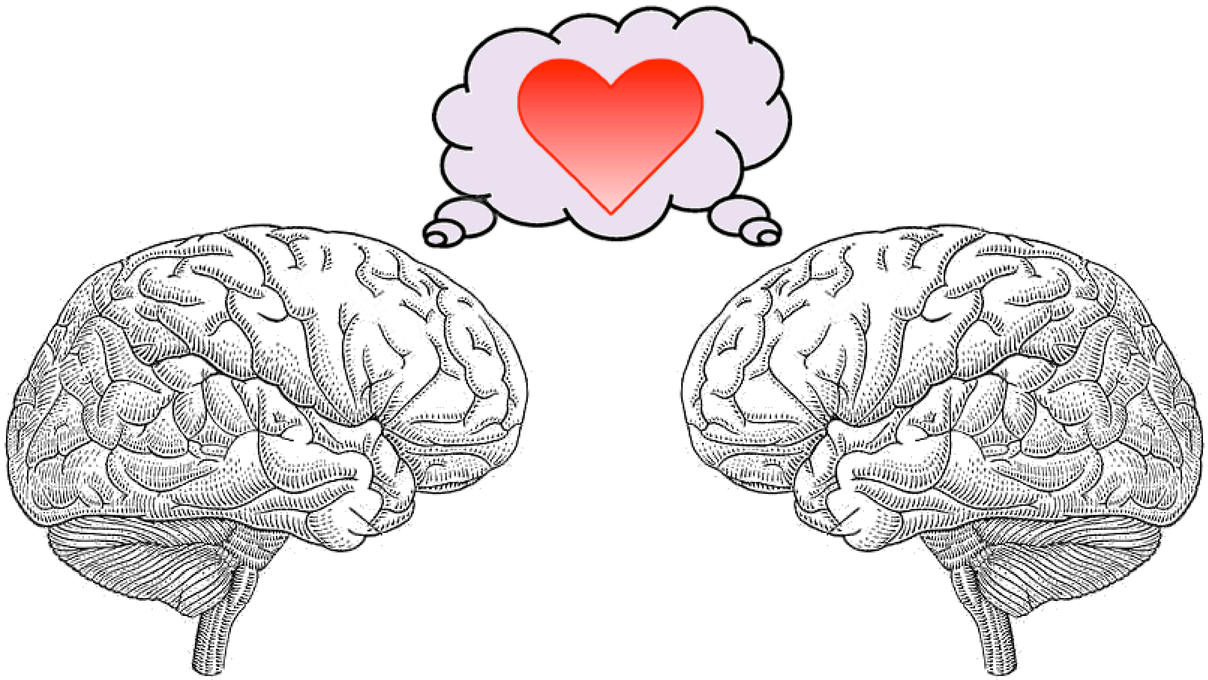
"The most complex system in the known universe."



Being and Value: Toward a Constructive Postmodern Metaphysics, by Frederick Ferre (1996)

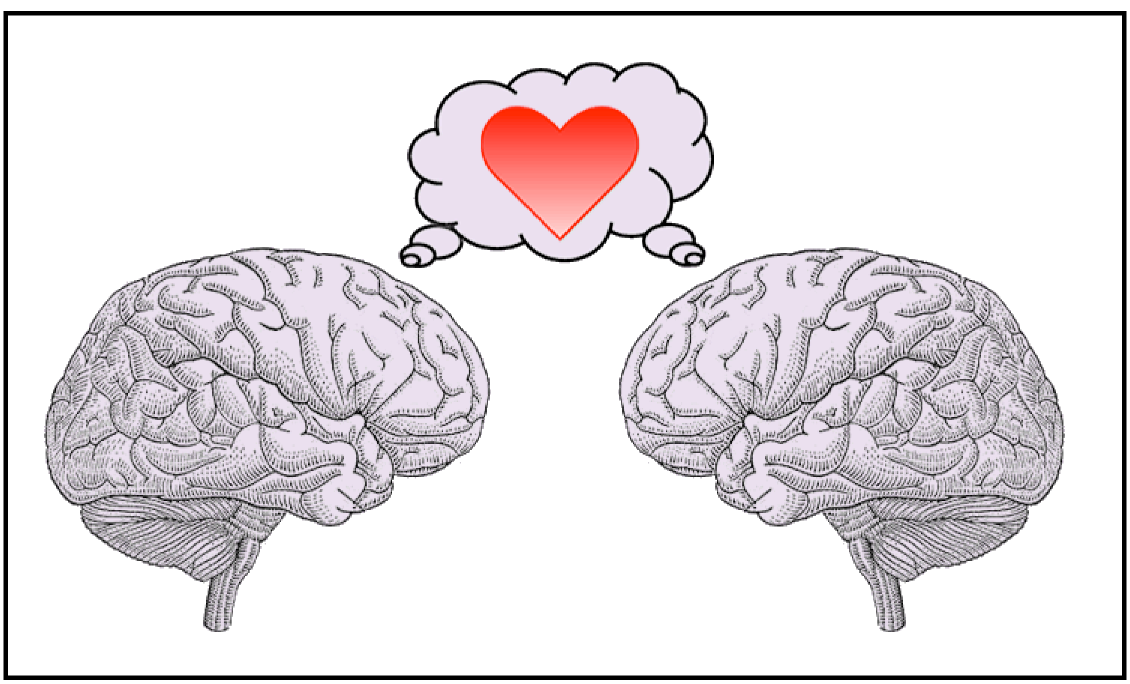
11_brains.psd

Infinitely More Complex than the Human Brain:
Two Brains in Love



12_2brains.psd

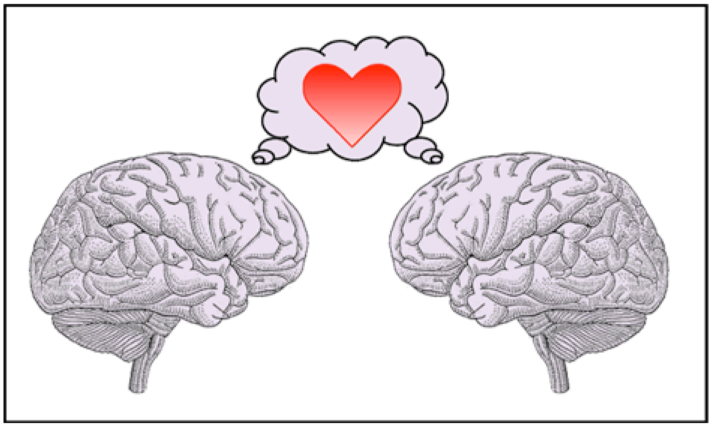
Quantify a System's Dynamics:
Step #1: *"Draw a distinction"*

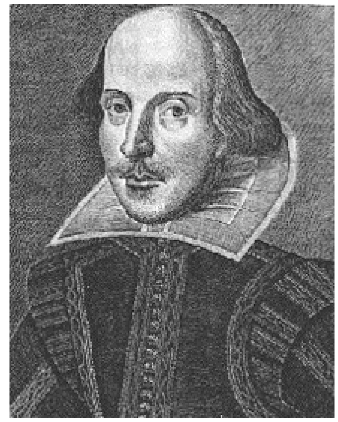


Laws of Form, by George Spencer-Brown (1969)

13_distinction.psd

Quantify a System's Dynamics:
Step #2: *Specify an example*





Romeo and Juliet, by William Shakespeare (1594)

Strogatz, S. H. 1988. Love affairs and differential equations. *Mathematics Magazine* 61:35

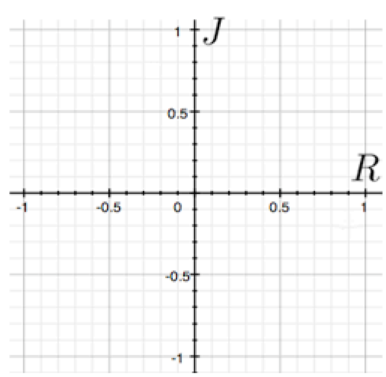
14_specify.psd

Quantify a System's Dynamics:
Step #3: *Quantify the salient features*

<u>The Dynamics of Romeo and Juliet</u>

Romeo's feelings for Juliet: $R(t)$

Juliet's feelings for Romeo: $J(t)$



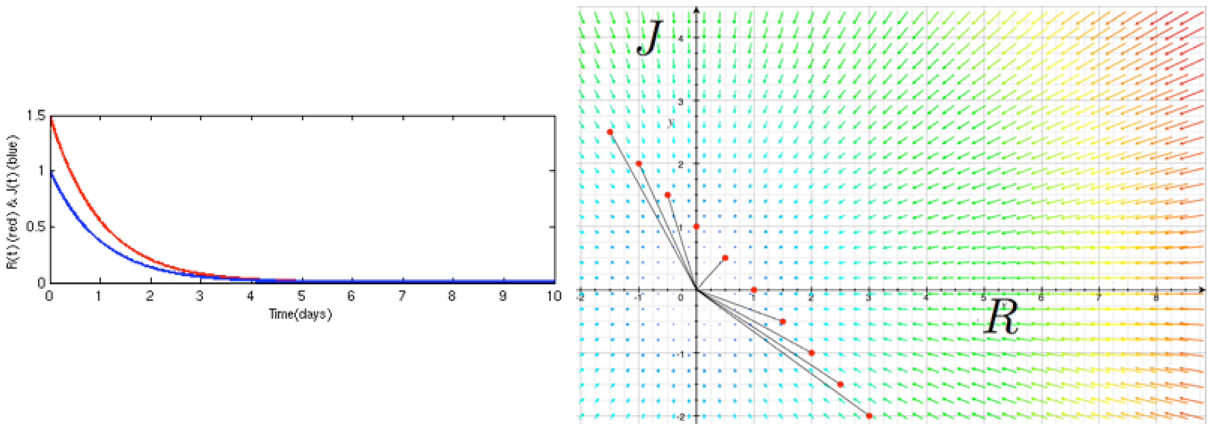
Strogatz, S. H. 1988. Love affairs and differential equations. *Mathematics Magazine* 61:35

15_quantify.psd

The Dynamics of Romeo and Juliet

Case #1: Independent dreamers
Drift towards indifference

$$\frac{d{\color{red}R}(t)}{dt} = -{\color{red}R}(t) \qquad \frac{d{\color{blue}J}(t)}{dt} = -{\color{blue}J}(t)$$

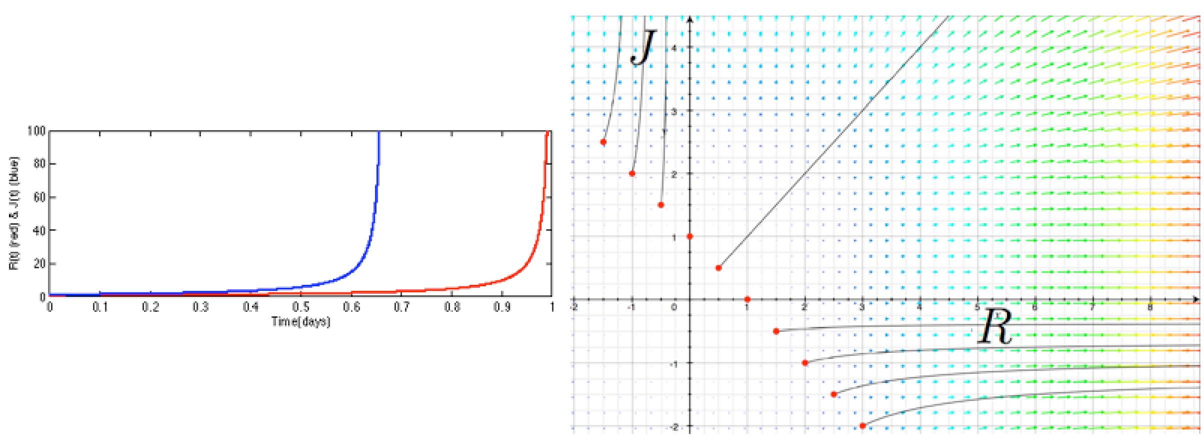


16_RJ1_indifference.psd

The Dynamics of Romeo and Juliet

Case #1: Independent dreamers
"Absence makes the heart grow fonder"

$$\frac{d{\color{red}R}(t)}{dt} = {\color{red}R}(t)^2 \qquad \frac{d{\color{blue}J}(t)}{dt} = {\color{blue}J}(t)^2$$



17_RJ1_absence.psd

Digression: Solving differential equations

$$\frac{dx}{dt} \equiv \dot{x} = f(x)g(t)$$

Linear equations

$$\begin{aligned} \dot{x} &= ax \\ \frac{dx}{x} &= a\,dt \\ \ln(x) &= at + c \\ x(t) &= e^{at+c} = x(0)e^{at} \end{aligned}$$

Quadratic equations

$$\begin{aligned} \dot{x} &= x^2 \\ \frac{dx}{x^2} &= dt \\ -\frac{1}{x} &= t + c \\ x(t) &= \frac{1}{c - t} \end{aligned}$$

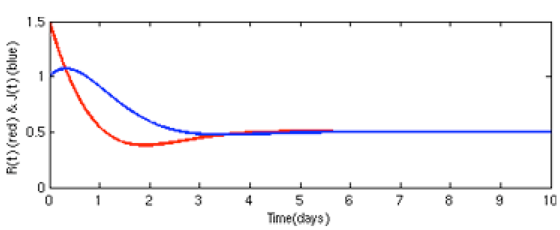
18_solve.psd

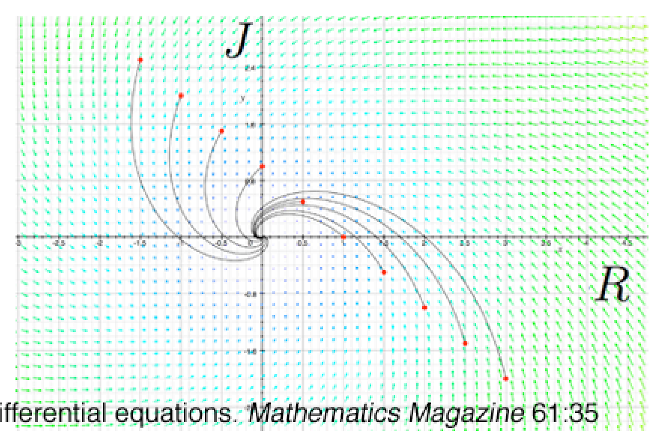
The Dynamics of Romeo and Juliet

Case #2: Two to Tango

Fickle Romeo and Juliet

$$\begin{aligned} \frac{dR(t)}{dt} &= -R(t) - J(t) \\ \frac{dJ(t)}{dt} &= -J(t) + R(t) \end{aligned}$$





Strogatz, S. H. 1988. Love affairs and differential equations. *Mathematics Magazine* 61:35

19_RJ2_fickle.psd

General Form of Linear Dynamical Equations

2-Dimensional dynamics

$$\frac{dR(t)}{dt} = a(R_0 - R(t)) - bJ(t)$$
$$\frac{dJ(t)}{dt} = d(J_0 - J(t)) + cR(t)$$

$$\frac{d}{dt}\begin{bmatrix} R(t) \\ J(t) \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} R_0 - R(t) \\ J_0 - J(t) \end{bmatrix}$$

Ordinary, homogenous, linear, first-order differential equation:

$$\boxed{\dot{x} = Ax}$$

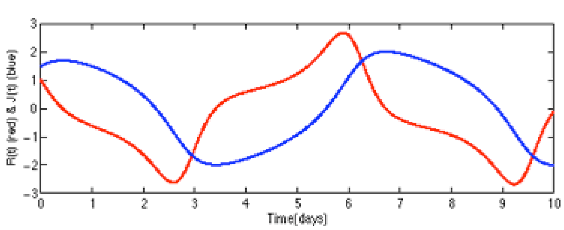
20_RJ2_matrix.psd

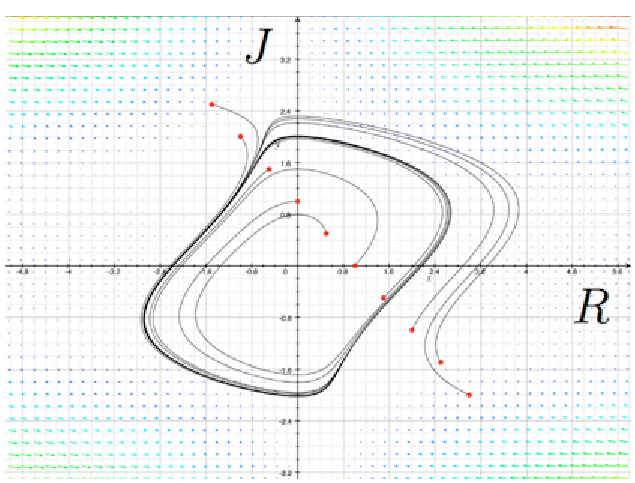
The Dynamics of Romeo and Juliet

Case #2b: Two to Tango

Fickle Romeo, modulated by the intensity of Juliet's feelings

$$\frac{dR(t)}{dt} = R(t)(1 - J(t)^2) - J(t)$$
$$\frac{dJ(t)}{dt} = R(t)$$





21_RJ2_oscil.psd

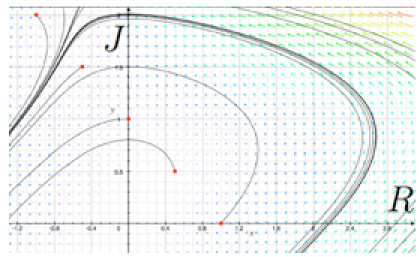
The Dynamics of Romeo and Juliet

Case #2b: Two to Tango

Fickle Romeo, modulated by the intensity of Juliet's feelings

$$\frac{dR(t)}{dt} = R(t)(1 - J(t)^2) - J(t)$$

$$\frac{dJ(t)}{dt} = R(t)$$



Eliminate Romeo to obtain second-order differential equation in Juliet:

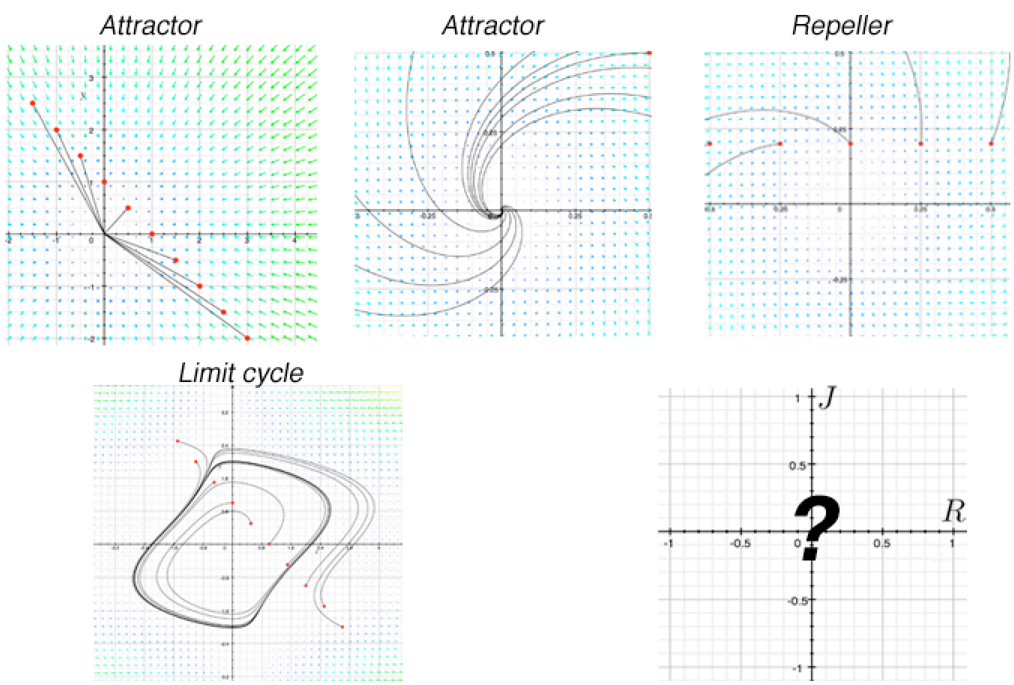
$$\frac{d^2 J(t)}{dt^2} = a\frac{dJ(t)}{dt}(1 - J(t)^2) - J(t)$$

Van der Pol oscillator yields a stable limit cycle.

22_RJ2_2ndOrder.psd

Quantify a System's Dynamics:
Step #4: *Classify the Dynamics*

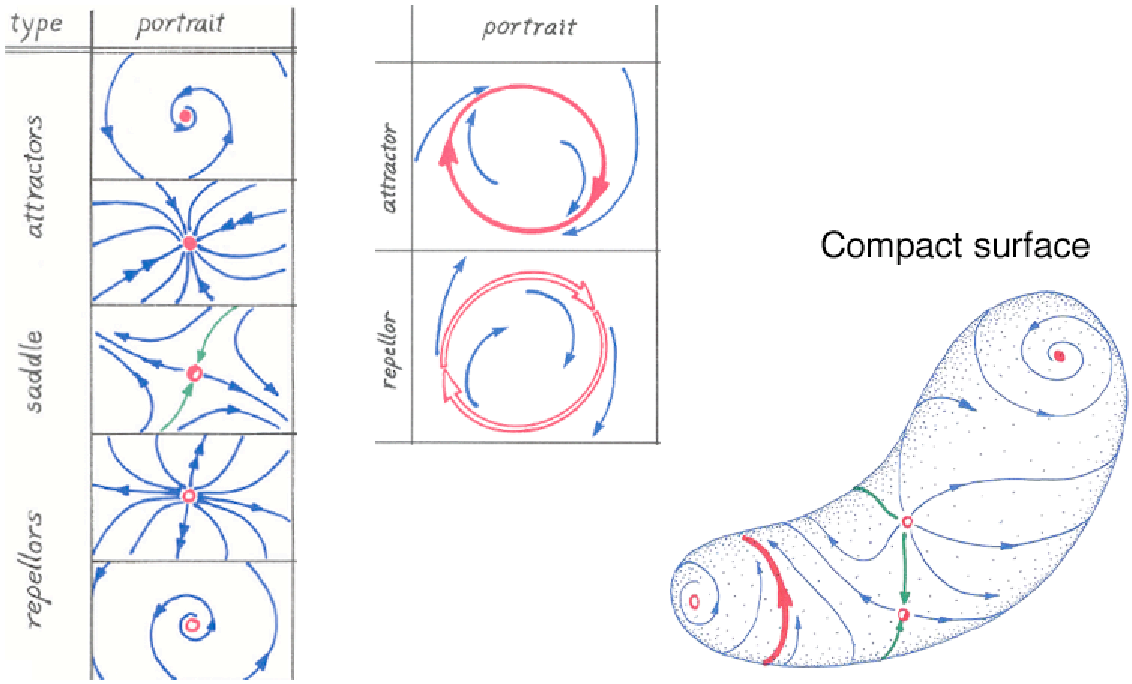
<u>The Dynamics of Romeo and Juliet</u>



23_RJ2_classify.psd

The Classification of 2-Dimensional Dynamics

Piexoto's Theorem

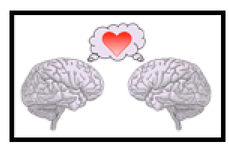


24_classif.psd

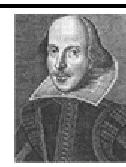
I. Dynamics: 2-Dimensional flow geometries

<u>Steps to quantify a System's Dynamics</u>:

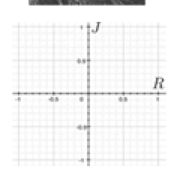
Step #1:*"Draw a distinction"*



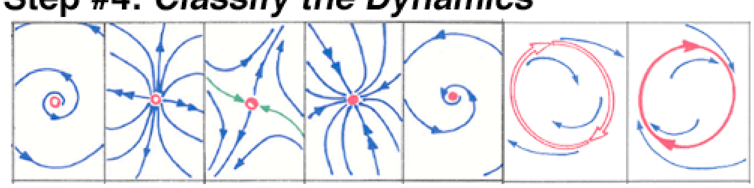
Step #2: *Specify an example*



Step #3: *Quantify the salient features*



Step #4: *Classify the Dynamics*



25_summary.psd